Paper presented at the
66th EAGE Conference & Exhibition
Paris, France, 6th-10th June 2004

Fast Imaging / Inversion of Synchronous Magnetotelluric and Controlled Source Data

Written by B. Sh. Singer and E. B. Fainberg

Fast Imaging / Inversion of Synchronous Magnetotelluric and Controlled Source Data

B. SH. SINGER[1] AND E. B. FAINBERG[2]
[1] CSIRO Petroleum, PO Box 1130, Bentley, WA 6102, Australia
[2] Geoelectromagnetic Research Institute, Russian Acad. Sci., Box 30, Troitsk, Moscow Region 142190, Russia

Abstract

Thin layer inversion algorithms allow for a fast determination of essential parameters of anomalous bodies. The algorithms are adjusted to expected geo-electric situation in the area of interest. Instead of fitting the medium parameters, the algorithms use a technique, the most time consuming part of which resembles spatial filtration of the electromagnetic field measured at the surface of the earth or at bottom of the sea. When used on a PC, a fraction of a minute is necessary to invert the data determined on a grid of 10^4 nodes. Examples of inversion are considered using electromagnetic data simulated for an environmental/mineral as well as mapping of gas hydrates types of problems. The approach can be used for interpretation of natural and controlled source measurements.

Introduction

Traditionally inversion of geophysical data is carried using a “trial and error” approach. The approach can be summarized as describing the model of interest with a limited set of parameters, inputting an initial guess for these parameters, numerical simulation of the response of thus defined model and corresponding Frechet derivatives, evaluation of the discrepancy between computed and measured responses, modification of the model parameters in the direction pointed out by Frechet derivatives, computation of the response and Frechet derivatives of the modified model, and so on until the desirable agreement between simulated and measured responses is achieved.

While, it is our opinion that this approach should be used at least at the final stage of any inversion, it is well known that straightforward application of “trials and errors” often encounters difficulties caused by the ill-posed character of the problem. In particular, the area of equivalence, i.e. the number of models that satisfy experimental data at the same level of accuracy, may be significant, especially, when dealing with complicated models requiring a large number of unknown parameters to be determined. In such circumstances, the initial guess may have a crucial effect on what models are finally chosen as the best fit. Thus, the use of a priori information of geological or geophysical origin becomes of extreme importance.

We believe that application of the thin layer approaches, some of which are outlined in this paper, may often appear sufficient for determining the most important parameters of the 3D geo-electric cross section. In other situation, the results may serve as an educated initial guess for a full “trial and error” approach, if possible, in combination with other independent data. An incentive for developing this type of inversion is the fact that, for instance, determining the conductivity cross section from the magnetotelluric impedance represents an ill-posed problem, while determining the conductance cross section does not (Dmitriev 1987). A practical realization of the thin sheet approach to inversion of 2D magnetovariational data was first reported by Schmucker (1971). Approaches to interpretation of 3D magnetotelluc data were considered by Vasseur and Weidelt (1977), Avdeyev, Fainberg, and Singer (1989), and Singer and Fainberg (1997).

Heterogeneous conductive layers

We consider the situation when lateral heterogeneities can be confined to one or several horizontal layers embedded into an otherwise laterally homogeneous formation. Assuming that a heterogeneous layer is more conductive than the surrounding formation and that the electromagnetic field penetrates the layer, it is possible to avoid considering the detail distribution of the electromagnetic field inside the layer. Instead, it may be requested for the solution of Maxwell's equations outside the layer to satisfy matching conditions at the horizontal planes confining the heterogeneities. For a conductive layer, the matching conditions are

$$\mathbf{E}_\tau - \mathbf{E}_\tau^- = \imath\omega\mu_0 h \mathbf{e}_z \times \mathbf{H}_\tau , \qquad \mathbf{H}_\tau - \mathbf{H}_\tau^- = S\,\mathbf{e}_z \times \mathbf{E}_\tau + h\nabla_\tau H_z \tag{1}$$

where subscript "τ" is used to denote the horizontal components of the electric field **E** and magnetic field **H**; h is the layer thickness, ω is the frequency of the electromagnetic field; $\mu_0 = 4\pi \cdot 10^{-7}\ H/m$; $\nabla_\tau = \mathbf{e}_x\partial_x + \mathbf{e}_y\partial_y$; $\mathbf{e}_x$ and $\mathbf{e}_y$ are horizontal, and $\mathbf{e}_z$ is the vertical unit vectors of the Cartesian coordinate system. In eq. (1), superscript "-" marks the components of the electromagnetic field that would be observed at the lower surface of the heterogeneous layer; corresponding components at the upper surface are left superscript free. In this approximation, conductive properties of the heterogeneous layer are described by its conductance

$$S(x,y) = \int_h \sigma(x,y,z)\,dz \ , \tag{2}$$

which depends only on the horizontal coordinates and is found by integration conductivity σ across the layer.

The applicability conditions of the thin layer approximation have been considered in a number of publications. As discussed by Singer & Fainberg (1999), the thin layer approximation requires variations of the electromagnetic field in time and space to be sufficiently slow for the field to penetrate this layer, i.e. $h << \min\{\lambda_S, \lambda_f\}$, where $\lambda_S = 2(\omega\mu_0 S)^{-1}$ is the skin depth of the electromagnetic field in rocks composing the layer, and λ_f is the characteristic length of field variations along the layer surface (accounting for wavelengths of the extraneous field, remoteness of the sharp boundaries, etc).

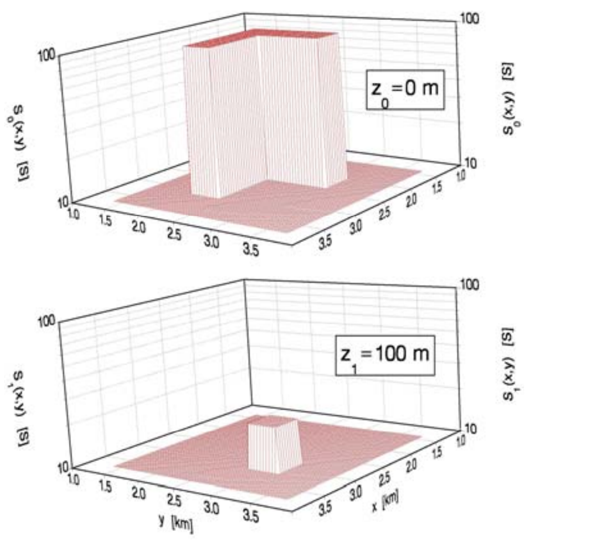

Figure 1. Conductance of two heterogeneous conductive layers

If electric and magnetic fields $\mathbf{E}_\tau$, **H** have been measured at the surface of the layer, then the first of matching conditions (1) can be used to find horizontal electric field $\mathbf{E}_\tau^-$ immediately underneath the heterogeneous layer. Assuming that the medium under the heterogeneous layer is laterally homogeneous and known, say, from a measurement made outside the anomalous area, the tangential magnetic field at the lower surface of the layer can be evaluated as $\mathbf{H}_\tau^- = \hat{Y} * (\mathbf{e}_z \times \mathbf{E}_\tau^-)$, where $\hat{Y}$ is the admittance tensor function of the underlying stratified medium (Dawson & Weaver 1979, Singer & Fainber 1997) and "*" denotes convolution with respect to horizontal coordinates. With $\mathbf{H}_\tau^-$ known, the second of matching conditions (1) represents an algebraic equation with respect to conductance S of the heterogeneous layer.

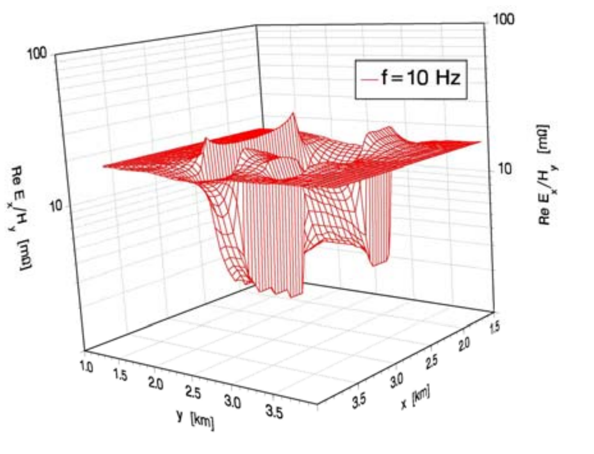

Figure 2. Apparent impedance for the model in Fig.2.

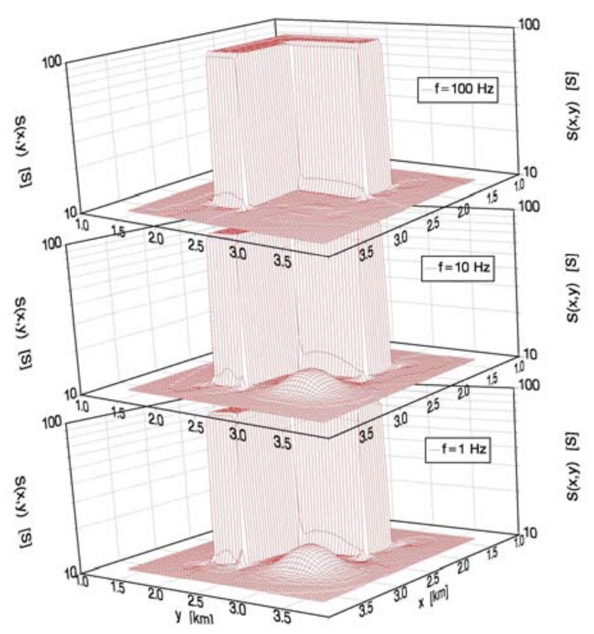

Figure 3. Inversion results at frequencies of 1, 10, and 100 *Hz*.

This approach can been used to invert the electromagnetic fields even if the geo-elelectric situation does not satisfy the assumption of anomalous bodies being confined to a single layer. For instance, we consider a model consisting of two heterogeneous layers of small thickness and apply an algorithm developed for a single layer model. In the model under consideration, the heterogeneous layers are embedded into a homogeneous half-space with resistivity of 20 Ωm. The surface heterogeneous layer of the model represents a large "corner-like" anomaly with conductance of 100 *S*, which by an order of magnitude exceeds that of the background. Another heterogeneous layer is located at the depth of 100 *m*. It has the same background conductance as the surface layer. The deep heterogeneous layer also includes geometrically small anomaly with conductance twice as large as that of the background (Fig. 1). The electromagnetic response of the model is numerically simulated for a plane wave source at frequencies of 1, 10, and 100 *Hz*. From Fig.2, the apparent impedance evaluated at the frequency of 10 *Hz* reflects the large subsurface anomaly but badly obscures its boundaries. Contribution of the smaller anomaly can hardly be detected at all. A similar situation is observed at other frequencies.

The simulated data are inverted using an algorithm implying that all heterogeneities are located at the surface. Inversion results are shown in three panels of Fig. 3. From the figure, the deep anomaly remains undetected at a higher frequency and reveals itself at the correct location as soon as the frequency is reduced. Somewhat reduced estimate of the conductance of the deep anomaly as well as smoothness of its boundaries are caused by the intentional use of the model that does not account for the fact that the second anomaly is located at a larger depth.

Heterogeneous resistive layers

In some situations the heterogeneous layer of interest is more resistive than the surrounding medium. For instance, areas of the so-called gas hydrate fields display a higher resistance compared to the surrounding sediments, which are also more resistive than the sea water (Yuan and Edwards 2001).

For a resistive heterogeneity considered a thin layer, the matching conditions must be modified to

$$\mathbf{E}_\tau - \mathbf{E}_\tau^- = -\nabla_\tau \left[T\, j_z\right] + \imath\omega\mu_0 h\, \mathbf{e}_z \times \mathbf{H}_\tau , \qquad \mathbf{H}_\tau - \mathbf{H}_\tau^- = h \nabla_\tau H_z , \tag{3}$$

where j_z is the vertical component of the current penetrating the heterogeneous layer, and

$$T(x,y) = \int_h \sigma^{-1}(x,y,z)\,dz \ , \tag{4}$$

is the transverse resistance of this layer. If the horizontal electric field is measured at the surface of the heterogeneous layer together with the magnetic field and the vertical component of the electric current (which can also be calculated from the magnetic field), then the following algorithm can be used to recover the distribution of the transverse resistance of the heterogeneous layer.

First magnetic field $\mathbf{H}_\tau^-$ is found from the second of matching (3). The electric field is then evaluated as $\mathbf{E}_\tau^- = -\hat{Z} * \left(\mathbf{e}_z \times \mathbf{H}_\tau^-\right)$, where $\hat{Z}$ is the impedance tensor function of the underlying stratified medium.

At the next step, the transverse resistance is found from the first of the matching conditions (3) reduced to

$$T\ j_z = T(x_0, y_0) j_z(x_0, y_0) - \int_{\gamma(x_0,y_0)} \{\mathbf{E}_\tau - \mathbf{E}_\tau^- - + \iota\omega\mu_0 h \mathbf{e}_z \times \mathbf{H}_\tau\} d\mathbf{l}\ , \qquad \textbf{(5)}$$

where x_0 and y_0 are coordinates of a point where either the transverse resistance is known or the vertical current is negligibly small; $\gamma(x_0,y_0)$.

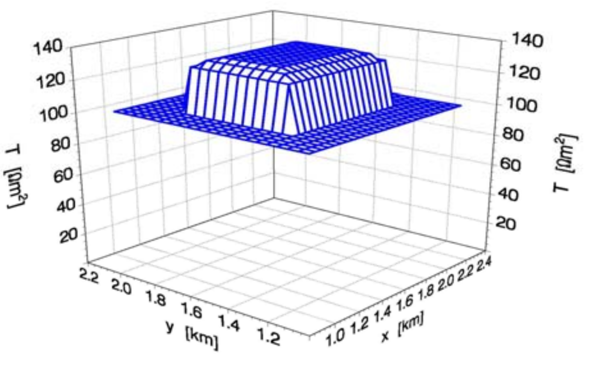

Figure 4. Distribution of transverse resistance in the model

The algorithm has been used to invert a simulated electromagnetic field in the model representing an ocean with the water conductivity of 3 *S/m*, and sediments with conductivity of 2 *S/m*, which are separated by a 100 *m* thick layer. Conductivity of sediments in the layer equals 1 *S/m*. The layer includes an anomalous body with the horizontal area of the size of 550 by 850 *m*. Conductivity inside the anomalous body is 0.75 *S/m*.

The electromagnetic field excited by a 600 *m* long vertical electric dipole suspended 200 *m* above the anomaly and is simulated using the 3D code by Singer et al. (2003). The frequency of the alternating current in the dipole equals 0.1 *Hz*. The distribution of the transverse resistance calculated using eq. (5) is shown in Fig. 5.

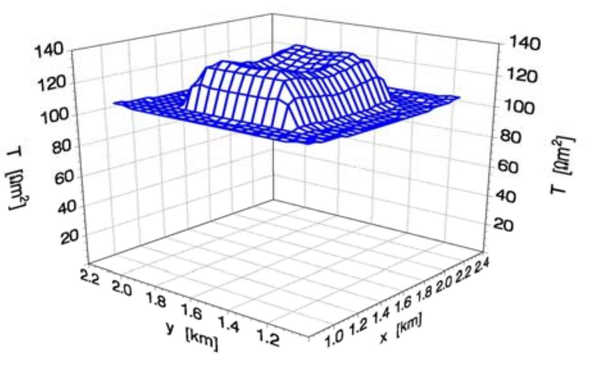

Figure 5. Transverse resistance from the inversion results.

Conclusions

The above consideration only outlines approaches based on thin layer models that are chosen to adequately represent the geo-electric situation in the area of interest. Measurements made with natural and controlled sources can be interpreted in a similar manner. The approach can be used in the frequency and time domains.

References

Avdeyev, D. B., Fainberg, E. B., and Singer, B. Sh., 1989. Thin sheet interpretation of alternating electromagnetic fields, *in*: Geolectrical investigations with a powerful current source on the Baltic shield, 228-235 (in Russian).

Dawson, T.W. and Weaver, J.T., 1979, Three-dimensional electromagnetic induction in a non-uniform thin sheet at the surface of uniformly conductive earth, *Geophys.J. R. astr.Soc.*, **59**, 445-462.

Dmitriev, V. I., 1987. Ill-posed problems of the electromagnetic methods in geophysics, *in*: Tikhonov, A. N. and Goncharsky, A. V. (eds), "Ill-posed problems of natural science", MGU, 54 -76 (in Russian).

Schmucker, U., 1971. Interpretation of induction anomalies above non-uniform surface layer, *Geophysics*, **36**, 156-165.

Singer, B. Sh. and Fainberg, E. B., 1997. Fast 3-D inversion algorithms, *Society of Exploration Geophysicists 67-th Annual meeting*, Expanded Abstract, 378-381.

Singer, B. Sh. & Fainberg, E. B., 1999. Modeling electromagnetic fields in thin heterogeneous layers with application to field generation by volcanoes: I - Theory and example. *Geophys. J. Int.*, **138**, 125-145.

Singer, B. Sh., Mezzatesta, A., and Wang, T., 2003, Integral equation approach based on contraction operators and Krylov space optimization, *in*: Macnae, J. and Liu, G.(eds), "Three-Dimensional Electromagnetics III", 26, 1- 14.

Vasseur, G. and Weidelt, P., 1977. Bimodal electromagnetic induction in non-uniform thin sheet with application to the northern Pyrenean anomaly, *Geophys. J. R. astr. Soc.*, **51**, 669-690.

Yuan, J. and Edwards, R. N., 2001. Tower seafloor electromagnetics and assessment of gas hydrate deposits, *Geophys. Res. Lett.*, **27**, 2397-2400.